PVP 14

Code: EE4T1

II B.Tech - II Semester – Regular/Supplementary Examinations – April 2017

COMPLEX VARIABLES & SPECIAL FUNCTIONS
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

PART – A

Answer ***all*** the questions. All questions carry equal marks

11 x 2 = 22

1.

a) Prove that f(z) = $\bar{z}$ is not analytic at any point.

b) Separate the real and imaginary parts of cot z.

c) Define analytic and entire functions.

d) Obtain the Taylor series expansion e^{1+z} in the powers of z – 1.

e) Evaluate $\int_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2)\,dz$ along $y=x^2$

f) Find the Residue $f(z) = \frac{z^3}{z^2 - 1}$ at z =1

g) Find the poles and residue at each pole of $\frac{z}{z^2 - 4}$

h) Define conformal transformation.

i) Find the region in the w – plane in which the rectangle bounded by the lines x = 0, y = 0 , x = 2 , and y = 1 is mapped under the transformation w = z + (2+3i)

j) Express $J_3(x)$ in terms of J_0 and J_1

k) Show that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$

PART – B

Answer any ***THREE*** questions. All questions carry equal marks.

3 x 16 = 48 M

2. a) If f(z) is an analytic function, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(z)|^2 .$$ 8 M

b) If $f(Z) = u + iv$ is an analytic function of Z and if

$u - v = e^x(\cos y - \sin y)$, find f(z) in terms of z. 8 M

3. a) Evaluate $\int_c \frac{z^2 - 2z - 2}{(z^2+1)^2 z}\,dz$ where c is $|z - i| = 1/2$ using Cauchy's integral formula. 8 M

b) Expand $\frac{1}{(z^2 - 3z + 2)}$ in the region

i) $0 < |z - 1| < 1$ ii) $1 < |z| < 2$. 8 M

4. a) i) Evaluate $\int_C \frac{(\sin \pi z^2 + \cos \pi z^2)\,dz}{(z-1)^2(z-2)}$ where c is the circle $|z| = 3$ using residue theorem. 4 M

ii) Find the poles $\frac{e^{iz}}{z^2+1}$ and corresponding residues. 4 M

b) Evaluate by residue theorem $\int_0^{2\pi} \frac{d\theta}{2 + Cos\ \theta}$. 8 M

5. a) Prove that the transformation w=sin z maps the families of lines x = constant and y = constant in to two families of confocal central conics. 8 M

b) Find the image of the infinite strip $o < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ 8 M

6. a) Show that $\frac{n}{x} J_n(x) - J_n'(x) = J_{n+1}(x)$ 8 M

b) Show that $\int_{-1}^{1} x^2 P_{n-1}(x) P_{n+1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ 8 M